Lecture 15

Strong Induction

- It assumes all previous $P(K)$'s are true
- That is :

$$\left(\bigwedge_{i=1}^{K} P(K) \right) \rightarrow P(K+1)$$

Example Proving every $n \geq 2$ can be expressed as the the product of prime numbers.

Base Case: $n = 2$ $(2)(1) = 2$ ✓

Inductive Hypothesis: $2, 3, 4, 5, 6, \ldots, K-1, K$ all have prime factorization

Inductive Step: Cases:

Case1: $K+1$ is prime

Case2: $K+1 = a \cdot b$ when $a > 1$, $b > 1$

↳ Since $a > 1$ and $b > 1$ the $a < K+1$, $b < K+1$

↳ this means $1 < a < K+1$, $1 < b < K+1$

↳ our inductive hypothesis is that all n such that $2 \leq n \leq K$ have prime factorization

↳ So the product of a and b must have prime factorization since a has prime factorization and b has prime factorization

Pros and Cons of Induction

- Pros
  - Easy to prove some conjectures
  - Only need to look at two adjacent cases

- Cons
  - Cannot be used to explore new conjectures, we can only verify whether something is true.
  - The proof is "less intuitive"

- Importance
  - Allows you to better understand recursive algorithms.

Recursive Definitions + Structural Induction

- These are utilized by compilers to check the "grammar" of your code.
- A grammar is described by recursive definitions

Example of Recursion Def

- Consider the set S defined recursively as follows:
  - Base case: $3 \in S$
  - Recursive Step: If $x \in S$ and $y \in S$, then $x+y \in S$
- Prove S is set of all positive integers that are multiples of 3.

Proof:

$x \in S \cap y \in S \rightarrow \boxed{x+y} \in S$ $\quad$ $3k + 3j = 3(k+j) \in S$

$x = 3k$ $\quad$ $y = 3j$

- Consider the following recursively defined set S:
  1. $a \in S$
  2. If $x \in S$ then $(x) \in S$
- Prove by structural induction that every element in S contains an equal number of right and left parentheses.